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Vibration of functionally graded cylindrical shells based on different shear deformation shell theories with ring support under various boundary conditions †

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Abstract

In the present work, study of the vibration of thin cylindrical shells with ring supports made of a functionally gradient material (FGM) composed of stainless steel and nickel is presented. Material properties are graded in the thickness direction of the shell according to volume fraction power law distribution. Effects of boundary conditions and ring support on the natural frequencies of the FGM cylindrical shell are studied. The cylindrical shells have ring supports which are arbitrarily placed along the shell and which imposed a zero lateral deflection. The study is carried out using different shear deformation shell theories. The analysis is carried out using Hamilton's principle. The governing equations of motion of a FGM cylindrical shells are derived based on various shear deformation theories. Results are presented on the frequency characteristics, influence of ring support position and the influence of boundary conditions. The present analysis is validated by comparing results with those available in the literature.

Keywords: Vibration; Functionally gradient materials; Hamilton's principle; Cylindrical shell; Ring support

1. Introduction

Cylindrical shells have found many applications in industry. They are often used as load bearing structures for aircrafts, ships and buildings. The study of the vibration of cylindrical shells is an important aspect in their successful application.

The study of the free vibrations of cylindrical shells have been studied extensively. Among those who have studied the vibrations of cylindrical shells include Arnold and Warburton [1], Ludwig and Krieg [2], Chung [3], Soedel [4], Forsberg [5], Bhimaraddi [6], Soldatos and Hajigeoriou [7], Bert and Kumar [8], and Soldatos [9]. Recently, the present authors also presented studies on the influence of boundary conditions on the frequencies of a multi–layered cylindrical shell [10], and on the free vibrations of rotating cylindrical shells [11]. In all the above works, thin shell theories based on Love–hypothesis were used. Vibration of cylindrical shell with ring support is considered by Loy and Lam [12].

The concept of functionally gradient materials (FGMs) was first introduced in 1984 by a group of materials scientists in Japan, [13], as a means of preparing thermal barrier materials. Since then FGMs have attracted much interest as heat-shielding materials. An excellent collection of works on the vibration of cylindrical shells with thermal stresses and deformations can be found in [14, 15], and [16]. Najafizadeh and Isvandzibaei presented the vibration of functionally graded cylindrical shells based on higher order shear deformation plate theory with ring support [17].

Vibration study of the FGM shell structures is im-

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portant. However, study of the vibration of FGM shells with ring supports is limited. In this paper a study on the vibration of cylindrical shells with ring supports made of functionally gradient material (FGM) is presented. The functionally gradient material considered is composed of stainless steel and nickel where the volume fractions follow a power–law distribution. The analysis is carried out using Hamilton's principle. Studies are done for cylindrical shells with simply supported–simply supported SS–SS, clamped–clamped C-C, free–free F–F, clamped–simply supported C–SS, clamped–free C-F and free–simply supported F–SS boundary conditions with an arbitrary ring support along the axial direction of the cylindrical shell.

Results presented include the frequency characteristics of cylindrical shells with ring supports, the influence of ring support position and the influence of boundary conditions. The present analysis is examined by comparing results for functionally graded cylindrical shells without ring supports with others in the literature.

2. Functionally gradient materials

Consider a cylindrical shell of radius R, length L and thickness h made of FGM. For the cylindrical shell made of FGM the material properties such as the modulus of elasticity E, Poisson ratio ν and the mass density ρ are assumed to be functions of the volume fraction of the constituent materials when the coordinate axis across the shell thickness is denoted by z and measured from the shells middle plane and N is the power-law exponent, $0 \le N \le \infty$. The functional relationships between E, ν and ρ with z for a stainless steel and nickel FGM shell are assumed as [18],

$$E = E(z) = (E_1 - E_2)\left(\frac{2z+h}{2h}\right)^N + E_2 \tag{1}$$

$$\nu = \nu(z) = (\nu_1 - \nu_2)\left(\frac{2z+h}{2h}\right)^N + \nu_2 \tag{2}$$

$$\rho = \rho(z) = (\rho_1 - \rho_2)\left(\frac{2z+h}{2h}\right)^N + \rho_2 \tag{3}$$

From these equations, when $z = h/2$, $E = E_2$, $\nu = \nu_2$ and $\rho = \rho_2$, and when $z = -h/2$, $E = E_1$, $\nu = \nu_1$ and $\rho = \rho_1$. The material properties vary continuously from material 2 at the inner surface of the cylindrical shell to material 1 at the outer surface of the cylindrical shell. A cylindrical shell composed of functionally gradient material is essentially an inhomogeneous shell consisting of a mixture of isotropic materials. For such a shell, unlike shells composed of fiber-reinforced materials where transverse shear deformation effects can be significant because of the high elastic modulus to the transverse modulus ratio, if the thickness-to-radius ratio is not more than 0.05, classical thin shell theory is valid.

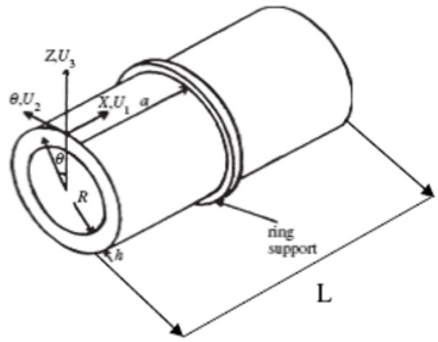

Fig. 1. Geometry of a cylindrical shell with ring support.

3. Strains-displacement relationships

The strain-displacement relationships for a thin shell are [19],

$$\in_{11} = \frac{1}{A_1(1+\frac{\alpha_3}{R_1})}\left[\frac{\partial U_1}{\partial \alpha_1} + \frac{U_2}{A_2}\frac{\partial A_1}{\partial \alpha_2} + U_3\frac{A_1}{R_1}\right] \tag{4}$$

$$\in_{22} \frac{1}{A_2(1+\frac{\alpha_3}{R_2})}\left[\frac{\partial U_2}{\partial \alpha_2} + \frac{U_1}{A_1}\frac{\partial A_2}{\partial \alpha_1} + U_3\frac{A_2}{R_2}\right] \tag{5}$$

$$\in_{33} = \frac{\partial U_3}{\partial \alpha_3} \tag{6}$$

$$\in_{12} = \frac{A_1(1+\frac{\alpha_3}{R_1})}{A_2(1+\frac{\alpha_3}{R_2})}\frac{\partial}{\partial \alpha_2}\left(\frac{U_1}{A_1(1+\frac{\alpha_3}{R_1})}\right) + \frac{A_2(1+\frac{\alpha_3}{R_2})}{A_1(1+\frac{\alpha_3}{R_1})}\frac{\partial}{\partial \alpha_1}\left(\frac{U_2}{A_2(1+\frac{\alpha_3}{R_2})}\right) \tag{7}$$

$$\in_{13} = A_1(1+\frac{\alpha_3}{R_1})\frac{\partial}{\partial \alpha_3}\left(\frac{U_1}{A_1(1+\frac{\alpha_3}{R_1})}\right) + \frac{1}{A_1(1+\frac{\alpha_3}{R_1})}\frac{\partial U_3}{\partial \alpha_1} \tag{8}$$

$$\in_{23} = A_2(1+\frac{\alpha_3}{R_2})\frac{\partial}{\partial \alpha_3}\left(\frac{U_2}{A_2(1+\frac{\alpha_3}{R_2})}\right) + \frac{1}{A_2(1+\frac{\alpha_3}{R_2})}\frac{\partial U_3}{\partial \alpha_2} \tag{9}$$

$$A_1 = \left|\frac{\partial \bar{r}}{\partial \alpha_1}\right|, \quad A_2 = \left|\frac{\partial \bar{r}}{\partial \alpha_2}\right| \tag{10}$$

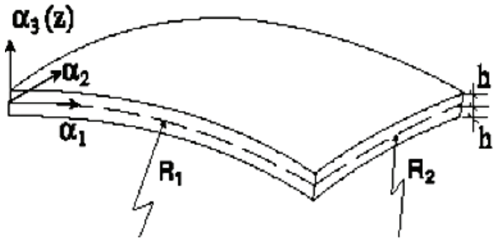

Fig. 2. Geometry of a generic shell.

where A_1 and A_2 are the fundamental form parameters or Lame parameters, U_1, U_2 and U_3 are the displacement at any point ($\alpha_1, \alpha_2, \alpha_3$), R_1 and R_2 are the radius of curvature related to α_1, α_2 and α_3 respectively.

The third-order theory of Reddy [20] used in the present study is based on the following displacement field

$$\begin{cases} U_1 = u_1(\alpha_1,\alpha_2) + \alpha_3.\phi_1(\alpha_1,\alpha_2) + \alpha_3^2.\psi_1(\alpha_1,\alpha_2) + \alpha_3^3.\beta_1(\alpha_1,\alpha_2) \\ U_2 = u_2(\alpha_1,\alpha_2) + \alpha_3.\phi_2(\alpha_1,\alpha_2) + \alpha_3^2.\psi_2(\alpha_1,\alpha_2) + \alpha_3^3.\beta_2(\alpha_1,\alpha_2) \\ U_3 = u_3(\alpha_1,\alpha_2) \end{cases} \tag{11}$$

These equations can be reduced by satisfying the stress-free conditions on the top and bottom faces of the laminates, which are equivalent to $\in_{13} = \in_{23} = 0$ at $z = \pm\frac{h}{2}$ Thus

$$\begin{cases} U_1 = u_1(\alpha_1,\alpha_2) + \alpha_3.\phi_1(\alpha_1,\alpha_2) - C_1.\alpha_3^3(-\frac{u_1}{R_1} + \phi_1 + \frac{\partial u_3}{A_1\partial\alpha_1}) \\ U_2 = u_2(\alpha_1,\alpha_2) + \alpha_3.\phi_2(\alpha_1,\alpha_2) - C_1.\alpha_3^3(-\frac{u_2}{R_2} + \phi_2 + \frac{\partial u_3}{A_2\partial\alpha_2}) \\ U_3 = u_3(\alpha_1,\alpha_2) \end{cases} \tag{12}$$

where $C_1 = \frac{4}{3h^2}$. Substituting Eq. (12) into nonlinear strain-displacement relation (4)-(9), we can obtain for the third-order theory of Reddy

$$\begin{Bmatrix} \in_{11} \\ \in_{22} \\ \in_{12} \end{Bmatrix} = \begin{Bmatrix} \in_{11}^0 \\ \in_{22}^0 \\ \in_{12}^0 \end{Bmatrix} + \alpha_3 \begin{Bmatrix} k_{11} \\ k_{22} \\ k_{12} \end{Bmatrix} + \alpha_3^3 \begin{Bmatrix} k'_{11} \\ k'_{22} \\ k'_{12} \end{Bmatrix} \tag{13}$$

$$\begin{Bmatrix} \in_{13} \\ \in_{23} \end{Bmatrix} = \begin{Bmatrix} \gamma_{13}^0 \\ \gamma_{23}^0 \end{Bmatrix} + \alpha_3^2 \begin{Bmatrix} \gamma_{13}^2 \\ \gamma_{23}^2 \end{Bmatrix} + \alpha_3^3 \begin{Bmatrix} \gamma_{13}^3 \\ \gamma_{23}^3 \end{Bmatrix} \tag{14}$$

Where

$$\begin{Bmatrix} \in_{11}^0 \\ \in_{22}^0 \\ \in_{12}^0 \end{Bmatrix} = \begin{Bmatrix} (\frac{1}{A_1}\frac{\partial u_1}{\partial\alpha_1} + \frac{u_2}{A_1A_2}\frac{\partial A_1}{\partial\alpha_2} + \frac{u_3}{R_1}) \\ (\frac{1}{A_2}\frac{\partial u_2}{\partial\alpha_2} + \frac{u_1}{A_1A_2}\frac{\partial A_2}{\partial\alpha_1} + \frac{u_3}{R_2}) \\ \frac{A_2}{A_1}\frac{\partial}{\partial\alpha_1}(\frac{u_2}{A_2}) + \frac{A_1}{A_2}\frac{\partial}{\partial\alpha_2}(\frac{u_1}{A_1}) \end{Bmatrix} \tag{15}$$

$$\begin{Bmatrix} k_{11} \\ k_{22} \\ k_{12} \end{Bmatrix} = \begin{Bmatrix} (\frac{1}{A_1}\frac{\partial\phi_1}{\partial\alpha_1} + \frac{\phi_2}{A_1A_2}\frac{\partial A_1}{\partial\alpha_2}) \\ (\frac{1}{A_2}\frac{\partial\phi_2}{\partial\alpha_2} + \frac{\phi_1}{A_1A_2}\frac{\partial A_2}{\partial\alpha_1}) \\ (\frac{A_2}{A_1}\frac{\partial}{\partial\alpha_1}(\frac{\phi_2}{A_2}) + \frac{A_1}{A_2}\frac{\partial}{\partial\alpha_2}(\frac{\phi_1}{A_1})) \end{Bmatrix} \tag{16}$$

$$\begin{Bmatrix} k'_{11} \\ k'_{22} \\ k'_{12} \end{Bmatrix} = -C_1 \begin{Bmatrix} (\frac{1}{A_1}(-\frac{\partial u_1}{R_1\partial\alpha_1} + \frac{\partial\phi_1}{\partial\alpha_1} + \frac{\partial^2 u_3}{A_1\partial\alpha_1^2} - \frac{\partial A_1}{A_1^2\partial\alpha_1}\frac{\partial u_3}{\partial\alpha_1}) + \frac{\partial A_1}{\partial\alpha_2}\frac{1}{A_1A_2}(-\frac{u_2}{R_2} + \phi_2 + \frac{\partial u_3}{A_2\partial\alpha_2})) \\ (\frac{1}{A_2}(-\frac{\partial u_2}{R_2\partial\alpha_2} + \frac{\partial\phi_2}{\partial\alpha_2} + \frac{\partial^2 u_3}{A_2\partial\alpha_2^2} - \frac{\partial A_2}{A_2^2\partial\alpha_2}\frac{\partial u_3}{\partial\alpha_2}) + \frac{\partial A_2}{\partial\alpha_1}\frac{1}{A_1A_2}(-\frac{u_1}{R_1} + \phi_1 + \frac{\partial u_3}{A_1\partial\alpha_1})) \\ (\frac{A_2}{A_1}(-\frac{\partial}{R_2\partial\alpha_1}(\frac{u_2}{A_2}) + \frac{\partial}{\partial\alpha_1}(\frac{\phi_2}{A_2}) + \frac{1}{A_2^2}\frac{\partial^2 u_3}{\partial\alpha_1\partial\alpha_2} - \frac{1}{A_2^4}\frac{\partial A_2^2}{\partial\alpha_1}\frac{\partial u_3}{\partial\alpha_2}) + \\ + \frac{A_1}{A_2}(-\frac{\partial}{R_1\partial\alpha_2}(\frac{u_1}{A_1}) + \frac{\partial}{\partial\alpha_2}(\frac{\phi_1}{A_1}) + \frac{1}{A_1^2}\frac{\partial^2 u_3}{\partial\alpha_1\partial\alpha_2} - \frac{1}{A_1^4}\frac{\partial A_1^2}{\partial\alpha_2}\frac{\partial u_3}{\partial\alpha_1})) \end{Bmatrix} \tag{17}$$

$$\begin{Bmatrix} \gamma_{13}^0 \\ \gamma_{23}^0 \end{Bmatrix} = \begin{Bmatrix} (\phi_1 - \frac{u_1}{R_1} + \frac{1}{A_1}\frac{\partial u_3}{\partial\alpha_1}) \\ (\phi_2 - \frac{u_2}{R_2} + \frac{1}{A_2}\frac{\partial u_3}{\partial\alpha_2}) \end{Bmatrix},$$

$$\begin{Bmatrix} \gamma_{13}^2 \\ \gamma_{23}^2 \end{Bmatrix} = 3C_1 \begin{Bmatrix} (-\frac{u_1}{R_1} + \phi_1 + \frac{\partial u_3}{A_1\partial\alpha_1}) \\ (-\frac{u_2}{R_2} + \phi_2 + \frac{\partial u_3}{A_2\partial\alpha_2}) \end{Bmatrix} \tag{18}$$

$$\begin{Bmatrix} \gamma_{13}^3 \\ \gamma_{23}^3 \end{Bmatrix} = C_1 \begin{Bmatrix} \frac{(-\frac{u_1}{R_1} + \phi_1 + \frac{\partial u_3}{A_1\partial\alpha_1})}{R_1} \\ \frac{(-\frac{u_2}{R_2} + \phi_2 + \frac{\partial u_3}{A_2\partial\alpha_2})}{R_2} \end{Bmatrix} \tag{19}$$

where (ε^0, γ^0) are the membrane strains and ($k, k', \gamma^2, \gamma^3$) are the bending strains, known as the curvatures. Substituting $C_1 = 0$ into Eq. (12) we get

$$\begin{cases} U_1 = u_1(\alpha_1,\alpha_2) + \alpha_3.\phi_1(\alpha_1,\alpha_2) \\ U_2 = u_2(\alpha_1,\alpha_2) + \alpha_3.\phi_2(\alpha_1,\alpha_2) \\ U_3 = u_3(\alpha_1,\alpha_2) \end{cases} \tag{20}$$

where Eq. (20) is first-order theory of Reddy used in the present study which is based on the following displacement field. Substituting Eq. (20) into nonlinear strain-displacement relation (4)-(9), we can obtain for the first-order theory of Reddy

$$\begin{Bmatrix} \in_{11} \\ \in_{22} \\ \in_{12} \end{Bmatrix} = \begin{Bmatrix} \in_{11}^0 \\ \in_{22}^0 \\ \in_{12}^0 \end{Bmatrix} + \alpha_3 \begin{Bmatrix} k_{11} \\ k_{22} \\ k_{12} \end{Bmatrix} \tag{21}$$

$$\begin{Bmatrix} \in_{13} \\ \in_{23} \end{Bmatrix} = \begin{Bmatrix} \gamma_{13}^0 \\ \gamma_{23}^0 \end{Bmatrix} \tag{22}$$

Where

$$\begin{Bmatrix} \in_{11}^0 \\ \in_{22}^0 \\ \in_{12}^0 \end{Bmatrix} = \begin{Bmatrix} (\frac{1}{A_1}\frac{\partial u_1}{\partial \alpha_1} + \frac{u_2}{A_1 A_2}\frac{\partial A_1}{\partial \alpha_2} + \frac{u_3}{R_1}) \\ (\frac{1}{A_2}\frac{\partial u_2}{\partial \alpha_2} + \frac{u_1}{A_1 A_2}\frac{\partial A_2}{\partial \alpha_1} + \frac{u_3}{R_2}) \\ \frac{A_2}{A_1}\frac{\partial}{\partial \alpha_1}(\frac{u_2}{A_2}) + \frac{A_1}{A_2}\frac{\partial}{\partial \alpha_2}(\frac{u_1}{A_1}) \end{Bmatrix} \tag{23}$$

$$\begin{Bmatrix} k_{11} \\ k_{22} \\ k_{12} \end{Bmatrix} = \begin{Bmatrix} (\frac{1}{A_1}\frac{\partial \phi_1}{\partial \alpha_1} + \frac{\phi_2}{A_1 A_2}\frac{\partial A_1}{\partial \alpha_2}) \\ (\frac{1}{A_2}\frac{\partial \phi_2}{\partial \alpha_2} + \frac{\phi_1}{A_1 A_2}\frac{\partial A_2}{\partial \alpha_1}) \\ (\frac{A_2}{A_1}\frac{\partial}{\partial \alpha_1}(\frac{\phi_2}{A_2}) + \frac{A_1}{A_2}\frac{\partial}{\partial \alpha_2}(\frac{\phi_1}{A_1})) \end{Bmatrix} \tag{24}$$

$$\begin{Bmatrix} \gamma_{13}^0 \\ \gamma_{23}^0 \end{Bmatrix} = \begin{Bmatrix} (\phi_1 - \frac{u_1}{R_1} + \frac{1}{A_1}\frac{\partial u_3}{\partial \alpha_1}) \\ (\phi_2 - \frac{u_2}{R_2} + \frac{1}{A_2}\frac{\partial u_3}{\partial \alpha_2}) \end{Bmatrix} \tag{25}$$

where $(\varepsilon^0, \gamma^0)$ are the membrane strains and k is the bending strain, known as the curvatures.

4. Stress-strain relationships

Consider a cylindrical shell with internal ring support as shown in Fig. 1. R is the radius, L is the length, h is the thickness and a , is the position of the ring support along the axial direction of the cylindrical shell. The reference surface is chosen to be the middle surface of the cylindrical shell where an orthogonal coordinate system x,θ,z is fixed. The deformations of the shell with reference to this coordinate system are denoted by U_1, U_2 and U_3 in the x,θ and z directions, respectively. For a thin cylindrical shell, the stress-strain relationship are defined as,

$$\begin{Bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{23} \\ \sigma_{13} \\ \sigma_{12} \end{Bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & 0 & 0 & 0 \\ Q_{12} & Q_{22} & 0 & 0 & 0 \\ 0 & 0 & Q_{44} & 0 & 0 \\ 0 & 0 & 0 & Q_{55} & 0 \\ 0 & 0 & 0 & 0 & Q_{66} \end{bmatrix} \begin{Bmatrix} \in_{11} \\ \in_{22} \\ \in_{23} \\ \in_{13} \\ \in_{12} \end{Bmatrix} \tag{26}$$

For an isotropic cylindrical shell the reduced stiffness Q_{ij} (i , j=1, 2 and 6) is defined as

$$Q_{11} = Q_{22} = \frac{E}{1-\nu^2} \tag{27}$$

$$Q_{12} = \frac{\nu E}{1-\nu^2} \tag{28}$$

$$Q_{44} = Q_{55} = Q_{66} = \frac{E}{2(1+\nu)} \tag{29}$$

where E is Young's modulus and ν is Poisson's ratio. Defining

$$\begin{aligned} &\left\{A_{ij}, B_{ij}, D_{ij}, E_{ij}, F_{ij}, G_{ij}, H_{ij}\right\} \\ &= \int_{-h/2}^{h/2} Q_{ij} \left\{1, \alpha_3, \alpha_3^2, \alpha_3^3, \alpha_3^4, \alpha_3^5, \alpha_3^6\right\} d\alpha_3 \end{aligned} \tag{30}$$

where Q_{ij} are functions of z for functionally gradient materials. Here A_{ij} denote the extensional stiffness, D_{ij} the bending stiffness, B_{ij} the bending-extensional coupling stiffness and $E_{ij}, F_{ij}, G_{ij}, H_{ij}$ are the extensional, bending, coupling, and higher-order stiffness.

5. The stress resultants

For a thin cylindrical shell the force and moment results are defined as

$$\begin{Bmatrix} N_{11} \\ N_{22} \\ N_{12} \end{Bmatrix} = \int_{-\frac{h}{2}}^{\frac{h}{2}} \begin{Bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{12} \end{Bmatrix} d\alpha_3 , \quad \begin{Bmatrix} M_{11} \\ M_{22} \\ M_{12} \end{Bmatrix} = \int_{-\frac{h}{2}}^{\frac{h}{2}} \begin{Bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{12} \end{Bmatrix} \alpha_3^3 d\alpha_3 \tag{31}$$

$$\begin{Bmatrix} P_{11} \\ P_{22} \\ P_{12} \end{Bmatrix} = \int_{-\frac{h}{2}}^{\frac{h}{2}} \begin{Bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{12} \end{Bmatrix} \alpha_3^3 d\alpha_3 , \quad \begin{Bmatrix} P_{13} \\ \\ P_{23} \end{Bmatrix} = \int_{-\frac{h}{2}}^{\frac{h}{2}} \begin{Bmatrix} \sigma_{13} \\ \\ \sigma_{23} \end{Bmatrix} \alpha_3^3 d\alpha_3 \tag{32}$$

$$\begin{Bmatrix} Q_{13} \\ Q_{23} \end{Bmatrix} = \int_{-\frac{h}{2}}^{\frac{h}{2}} \begin{Bmatrix} \sigma_{13} \\ \sigma_{23} \end{Bmatrix} d\alpha_3 , \quad \begin{Bmatrix} R_{13} \\ R_{23} \end{Bmatrix} = \int_{-\frac{h}{2}}^{\frac{h}{2}} \begin{Bmatrix} \sigma_{13} \\ \sigma_{23} \end{Bmatrix} \alpha_3{}^2 d\alpha_3 \tag{33}$$

6. The equations of motion for vibration of a generic shell

The equations of motion for vibration of a generic shell can be derived by using Hamilton's principle which is described by

$$\delta \int_{t_1}^{t_2} (\Pi - K) dt = 0 , \quad \Pi = U - V \tag{34}$$

where K, Π, U and V are the total kinetic, potential, strain and loading energies, t_1 and t_2 are arbitrary time. The kinetic, strain and loading energies of a cylindrical shell can be written as

$$K = \frac{1}{2} \int_{\alpha_1} \int_{\alpha_2} \int_{\alpha_3} \rho (\dot{U}_1^2 + \dot{U}_2^2 + \dot{U}_3^2) dV \tag{35}$$

$$U = \int_{\alpha_1} \int_{\alpha_2} \int_{\alpha_3} (\sigma_{11} \in_{11} + \sigma_{22} \in_{22} + \sigma_{12} \in_{12} + \sigma_{13} \in_{13} + \sigma_{23} \in_{23}) d\forall \tag{36}$$

$$V = \int_{\alpha_1} \int_{\alpha_2} (q_1 \delta U_1 + q_2 \delta U_2 + q_3 \delta U_3) A_1 A_2 d\alpha_1 d\alpha_2 \tag{37}$$

The infinitesimal volume is given by

$$dV = A_1 A_2 d\alpha_1 d\alpha_2 d\alpha_3 \tag{38}$$

with the use of Eqs. (11)-(19) and substituting into Eq. (34), we get the equations of motions a generic shell for the third-order theory of Reddy

$$\begin{aligned}
& -\frac{\partial (N_{11} A_2)}{\partial \alpha_1} + N_{22} \frac{\partial A_2}{\partial \alpha_1} - \frac{\partial (N_{12} A_1^2)}{A_1 \partial \alpha_2} - \frac{Q_{13}}{R_1} A_1 A_2 \\
& - \frac{\partial}{\partial \alpha_1} (\frac{P_{11} C_1 A_2}{R_1}) + \frac{P_{22} C_1}{R_1} \frac{\partial A_2}{\partial \alpha_1} - \frac{\partial}{\partial \alpha_2} (\frac{P_{12} C_1 A_1^2}{R_1}) \frac{1}{A_1} \\
& + \frac{3 C_1 R_{13}}{R_1} A_1 A_2 - \frac{C_1 P_{13} A_1 A_2}{R_1^2} \\
& = -(\ddot{u}_1 I_\circ + \ddot{\phi}_1 I_1 + \left[-C_1 (-\frac{\ddot{u}_1}{R_1} + \ddot{\phi} + \frac{\partial \ddot{u}_3}{A_1 \partial \alpha_1}) + \frac{C_1 \ddot{u}_1}{R_1} \right] I_3 \\
& \quad + \frac{C_1 \ddot{\phi}_1}{R_1} I_4 - \frac{C_1^2}{R_1} (-\frac{\ddot{u}_1}{R_1} + \ddot{\phi}_1 + \frac{\partial \ddot{u}_3}{A_1 \partial \alpha_1}) I_6)
\end{aligned} \tag{39}$$

$$\begin{aligned}
& \frac{\partial (N_{22} A_1)}{\partial \alpha_2} - N_{11} \frac{\partial A_1}{\partial \alpha_2} + \frac{\partial (N_{12} A_2^2)}{A_2 \partial \alpha_1} + \frac{Q_{23}}{R_2} A_1 A_2 \\
& + \frac{\partial}{\partial \alpha_2} (\frac{P_{22} C_1 A_1}{R_2}) - \frac{P_{11} C_1}{R_2} \frac{\partial A_1}{\partial \alpha_2} + \frac{\partial}{\partial \alpha_1} (\frac{P_{12} C_1 A_2^2}{R_2}) \frac{1}{A_2} \\
& - \frac{3 C_1 R_{23}}{R_2} A_1 A_2 + \frac{C_1 P_{23} A_1 A_2}{R_2^2} \\
& = (\ddot{u}_2 I_\circ + \ddot{\phi}_2 I_1 + \frac{C_1 \ddot{\phi}_2}{R_2} I_{42} \\
& \quad + \left[-c_1 (-\frac{\ddot{u}_2}{R_2} + \ddot{\phi}_2 + \frac{\partial \ddot{u}_3}{A_2 \partial \alpha_2}) + \frac{C_1 \ddot{u}_2}{R_2} \right] I_3 \\
& \quad - \frac{C_1^2}{R_2} (-\frac{\ddot{u}_2}{R_2} + \ddot{\phi}_2 + \frac{\partial u_3}{A_2 \partial \alpha_2}) I_6)
\end{aligned} \tag{40}$$

$$\begin{aligned}
& (-\frac{\partial^2 (P_{11} C_1 A_2 / A_1)}{\partial \alpha_1^2} + N_{11} \frac{A_1 A_2}{R_1} + \frac{\partial}{\partial \alpha_2} (\frac{C_1 P_{11}}{A_2} \frac{\partial A_1}{\partial \alpha_2}) \\
& + N_{22} \frac{A_1 A_2}{R_2} - \frac{\partial^2 (P_{22} A_1 C_1 / A_2)}{\partial \alpha_2^2} + \frac{\partial}{\partial \alpha_1} (\frac{P_{22} C_1}{A_1} \frac{\partial A_2}{\partial \alpha_1}) \\
& - \frac{\partial^2 (P_{12} C_1)}{\partial \alpha_1 \partial \alpha_2} - \frac{\partial}{\partial \alpha_2} (\frac{P_{12} C_1}{A_2^2} \frac{\partial A_2^2}{\partial \alpha_1}) - \frac{\partial^2 (P_{12} C_1)}{\partial \alpha_1 \partial \alpha_2} \\
& - \frac{\partial}{\partial \alpha_1} (\frac{P_{12} C_1}{A_1^2} \frac{\partial A_1^2}{\partial \alpha_2}) - \frac{\partial (Q_{13} A_2)}{\partial \alpha_1} + \frac{\partial (3 C_1 R_{13} A_2)}{\partial \alpha_1} \\
& - \frac{\partial}{\partial \alpha_1} (\frac{P_{13} C_1 A_2}{R_1}) - \frac{\partial (Q_{23} A_1)}{\partial \alpha_2} + \frac{\partial (3 C_1 R_{23} A_1)}{\partial \alpha_2} \\
& - \frac{\partial}{\partial \alpha_2} (\frac{C_1 P_{23} A_1}{R_2}) - \frac{\partial}{\partial \alpha_1} (\frac{P_{11} C_1 A_2}{A_1^2} \frac{\partial A_1}{\partial \alpha_1}) \\
& - \frac{\partial}{\partial \alpha_2} (P_{22} C_1 \frac{A_1}{A_2^2} \frac{\partial A_2}{\partial \alpha_2})) \\
& = -\{ \ddot{u}_3 I_\circ + C_1 \left[\frac{\partial}{\partial \alpha_1} (\frac{u_1}{A_1}) + \frac{\partial}{\partial \alpha_2} (\frac{u_2}{A_2}) \right] I_3 \\
& \quad + C_1 \left[\frac{\partial}{\partial \alpha_1} (\frac{\ddot{\phi}_1}{A_1}) + \frac{\partial}{\partial \alpha_2} (\frac{\ddot{\phi}_2}{A_2}) \right] I_4 - C_1^2 I_6 ((-\frac{\partial}{R_2 \partial \alpha_2} (\frac{\ddot{u}_2}{A_2}) \\
& \quad + \frac{\partial}{\partial \alpha_2} (\frac{\ddot{\phi}_2}{A_2}) + \frac{1}{A_2} \frac{\partial^2 \ddot{u}_3}{\partial \alpha_2^2} - \frac{\partial A_2}{A_1^2 \partial \alpha_2} \frac{\partial \ddot{u}_3}{\partial \alpha_2}) \\
& \quad + (-\frac{\partial}{R_1 \partial \alpha_1} (\frac{\ddot{u}_1}{A_1}) + \frac{\partial}{\partial \alpha_1} (\frac{\ddot{\phi}_1}{A_1}) + \frac{1}{A_1} \frac{\partial^2 \ddot{u}_3}{\partial \alpha_1^2} - \frac{\partial A_1}{A_1^2 \partial \alpha_1} \frac{\partial \ddot{u}_3}{\partial \alpha_1})) \}
\end{aligned} \tag{41}$$

$$\begin{aligned}
& -\frac{\partial (M_{11} A_2)}{\partial \alpha_1} + \frac{\partial (C_1 P_{11} A_2)}{\partial \alpha_1} + M_{22} \frac{\partial A_2}{\partial \alpha_1} - C_1 P_{22} \frac{\partial A_2}{\partial \alpha_1} \\
& - \frac{\partial (M_{12} A_1^2)}{A_1 \partial \alpha_2} + \frac{\partial (P_{12} C_1 A_1^2)}{A_1 \partial \alpha_2} - 3 C_1 R_{13} A_1 A_2 + A_1 A_2 Q_{13} \\
& + \frac{C_1 P_{13}}{R_1} A_1 A_2 = -\left[\ddot{u}_1 I_1 + \ddot{\phi}_1 I_2 - C_1 \ddot{u}_1 I_3 + (-2 C_1 \ddot{\phi}_1 + \right. \\
& \left. C_1 \frac{\ddot{u}_1}{R_1} - - \frac{C_1}{A_1} \frac{\partial \ddot{u}_3}{\partial \alpha_1}) I_4 + C_1^2 (-\frac{\ddot{u}_1}{R_1} + \ddot{\phi}_1 + \frac{\partial \ddot{u}_3}{A_1 \partial \alpha_1}) I_6 \right]
\end{aligned} \tag{42}$$

$$-\frac{\partial (M_{22} A_1)}{\partial \alpha_2} + \frac{\partial (C_1 A_1 P_{22})}{\partial \alpha_2} + M_{11} \frac{\partial A_1}{\partial \alpha_2} - C_1 P_{11} \frac{\partial A_1}{\partial \alpha_2}$$

$$-\frac{\partial(M_{12}A_2^2)}{A_2\partial\alpha_1}+\frac{\partial(P_{12}C_1A_2^2)}{A_2\partial\alpha_1}-3C_1R_{23}A_1A_2+A_1A_2Q_{23}$$
$$+\frac{C_1P_{23}}{R_2}A_1A_2=-\left[\ddot{u}_2I_1+\ddot{\phi}_2I_2-C_1\ddot{u}_2I_3+(-2C_1\ddot{\phi}_2\right.$$
$$\left.+C_1\frac{\ddot{u}_2}{R_2}-\frac{C_1}{A_2}\frac{\partial\ddot{u}_3}{\partial\alpha_2})I_4+C_1^2(\frac{\ddot{u}_2}{R_2}+\ddot{\phi}_2+\frac{\partial\ddot{u}_3}{A_2\partial\alpha_2})I_6\right]. \tag{43}$$

with the use of Eqs.(20)-(25) and substituting into Eq. (34), we get the equations of motion a generic shell for the first-order theory of Reddy

$$-\frac{\partial(N_{11}A_2)}{\partial\alpha_1}+N_{22}\frac{\partial A_2}{\partial\alpha_1}-\frac{\partial(N_{12}A_1^2)}{A_1\partial\alpha_2}-\frac{Q_{13}}{R_1}A_1A_2$$
$$=-[\ddot{u}_1I_\circ+\ddot{\phi}_1I_1] \tag{44}$$

$$\frac{\partial(N_{22}A_1)}{\partial\alpha_2}-N_{11}\frac{\partial A_1}{\partial\alpha_2}+\frac{\partial(N_{12}A_2^2)}{A_2\partial\alpha_1}+\frac{Q_{23}}{R_2}A_1A_2$$
$$=\ddot{u}_2I_\circ+\ddot{\phi}_2I_1 \tag{45}$$

$$N_{11}\frac{A_1A_2}{R_1}+N_{22}\frac{A_1A_2}{R_2}-\frac{\partial(Q_{13}A_2)}{\partial\alpha_1}-\frac{\partial(Q_{23}A_1)}{\partial\alpha_2}=-I_\circ\ddot{u}_3 \tag{46}$$

$$-\frac{\partial(M_{11}A_2)}{\partial\alpha_1}+M_{22}\frac{\partial A_2}{\partial\alpha_1}-\frac{\partial(M_{12}A_1^2)}{A_1\partial\alpha_2}+A_1A_2Q_{13}$$
$$=-\left[\ddot{u}_1I_1+\ddot{\phi}_1I_2\right] \tag{47}$$

$$-\frac{\partial(M_{22}A_1)}{\partial\alpha_2}+M_{11}\frac{\partial A_1}{\partial\alpha_2}-\frac{\partial(M_{12}A_2^2)}{A_2\partial\alpha_1}+A_1A_2Q_{23}$$
$$=-\left[\ddot{u}_2I_1+\ddot{\phi}_2I_2\right]. \tag{48}$$

For Eqs. (39)-(48) are defined as

$$I_i=\int_{-\frac{h}{2}}^{\frac{h}{2}}\rho\alpha_3^i d\alpha_3 \tag{49}$$

7. The equations of motion for vibration of a cylindrical shell

The curvilinear coordinates and fundamental form parameters for a cylindrical shell are

$$R_2=a\,,\ \frac{1}{R_1}=0\,,\ A_2=a\,,\ A_1=0\,,\ \alpha_3=\alpha_3\,,$$
$$\alpha_2=\theta\,,\ \alpha_1=x \tag{50}$$

Substituting relationship (50) into Eqs. (39)-(43) the equations of motions for vibration of cylindrical shell with the third-order theory of Reddy are converted to

$$a\frac{\partial N_{11}}{\partial x}+\frac{\partial N_{12}}{\partial\theta}=I_0\ddot{u}_1+(I_1-C_1I_3)\ddot{\phi}_1-C_1I_3\frac{\partial\ddot{u}_3}{\partial x} \tag{51}$$

$$\frac{\partial N_{22}}{\partial\theta}+C_1\frac{\partial P_{12}}{\partial x}+Q_{23}-3C_1R_{23}+C_1P_{23}$$
$$=(I_0+2\frac{C_1}{a}I_3+\frac{C_1^2}{a^2}I_6)\ddot{u}_2+(I_1-C_1I_3+\frac{C_1}{a}I_4-\frac{C_1^2}{a}I_6)\ddot{\phi}_2$$
$$-(\frac{C_1}{a}I_3-\frac{C_1^2}{a^2}I_6)\frac{\partial\ddot{u}_3}{\partial\theta} \tag{52}$$

$$-C_1a\frac{\partial^2P_{11}}{\partial x^2}+N_{22}-\frac{C_1}{a}\frac{\partial^2P_{22}}{\partial\theta^2}-2C_1\frac{\partial^2P_{12}}{\partial x\partial\theta}-a\frac{\partial Q_{13}}{\partial x}$$
$$+3C_1a\frac{\partial R_{13}}{\partial x}-\frac{\partial Q_{23}}{\partial\theta}++3C_1\frac{\partial R_{23}}{\partial\theta}-\frac{C_1}{a}\frac{\partial P_{23}}{\partial\theta}$$
$$=-C_1I_3\frac{\partial u_1}{\partial x}-\frac{C_1}{a}I_3\frac{\partial u_2}{\partial\theta}+(-C_1I_4+C_1^2I_6)\frac{\partial\ddot{\phi}_1}{\partial x}+$$
$$(-\frac{C_1}{a}I_4+\frac{C_1^2}{a}I_6)\frac{\partial\ddot{\phi}_2}{\partial\theta}-\frac{C_1^2}{a^2}I_6\frac{\partial\ddot{u}_2}{\partial\theta}+C_1^2I_6\frac{\partial^2\ddot{u}_3}{\partial x^2}$$
$$+\frac{C_1^2}{a}I_6\frac{\partial^2\ddot{u}_3}{\partial\theta^2}-\ddot{u}_3I_0 \tag{53}$$

$$-a\frac{\partial M_{11}}{\partial x}+C_1a\frac{\partial P_{11}}{\partial x}-\frac{\partial M_{12}}{\partial\theta}+C_1\frac{\partial P_{12}}{\partial\theta}-3C_1R_{13}a+aQ_{13}$$
$$=-I_1\ddot{u}_1+C_1I_3\ddot{u}_1+(-I_2+2C_1I_4-C_1^2I_6)\ddot{\phi}_1$$
$$+(C_1I_4-C_1^2I_6)\frac{\partial\ddot{u}_3}{\partial x} \tag{54}$$

$$-\frac{\partial M_{22}}{\partial\theta}-C_1\frac{\partial P_{22}}{\partial\theta}-a\frac{\partial M_{12}}{\partial x}+C_1a\frac{\partial P_{12}}{\partial x}-3C_1R_{23}a$$
$$+aQ_{23}+C_1R_{23}=(-I_1C_1I_3-\frac{C_1}{a}I_4)\ddot{u}_2$$
$$+(-I_2+2C_1I_4)\ddot{\phi}_2-\frac{C_1}{a}I_4\frac{\partial\ddot{u}_3}{\partial\theta} \tag{55}$$

Substituting relationship (50) into Eqs. (44)-(48) the equations of motions for vibration of cylindrical shell with the first-order theory of Reddy are converted to

$$a\frac{\partial N_{11}}{\partial x}+\frac{\partial N_{12}}{\partial\theta}=I_0\ddot{u}_1+I_1\ddot{\phi}_1 \tag{56}$$

$$\frac{\partial N_{22}}{\partial\theta}+Q_{23}=I_0\ddot{u}_2+I_1\ddot{\phi}_2 \tag{57}$$

$$N_{22}-a\frac{\partial Q_{13}}{\partial x}-\frac{\partial Q_{23}}{\partial\theta}=-\ddot{u}_3I_0 \tag{58}$$

$$-a\frac{\partial M_{11}}{\partial x}-\frac{\partial M_{12}}{\partial\theta}+aQ_{13}=-I_1\ddot{u}_1-I_2\ddot{\phi}_1 \tag{59}$$

$$-\frac{\partial M_{22}}{\partial\theta}-a\frac{\partial M_{12}}{\partial x}+aQ_{23}=-I_1\ddot{u}_2-I_2\ddot{\phi}_2 \tag{60}$$

8. Analysis

The displacement fields for a cylindrical shell with an arbitrary number of ring support and the displacement fields which satisfy these boundary conditions can be written as

$$\begin{aligned} u_1 &= \bar{A}\frac{\partial \phi(x)}{\partial x}\cos(n\theta)\cos(\omega t) \\ u_2 &= \bar{B}\phi(x)\sin(n\theta)\cos(\omega t) \\ u_3 &= \bar{C}\phi(x)\prod_{i=1}^{P}(x-a_i)^{\xi i}\cos(n\theta)\cos(\omega t) \\ \phi_1 &= \bar{D}\frac{\partial \phi(x)}{\partial x}\cos(n\theta)\cos(\omega t) \\ \phi_2 &= \bar{E}\phi(x)\sin(n\theta)\cos(\omega t) \end{aligned} \tag{61}$$

where, $\bar{A}$, $\bar{B}$, $\bar{C}$, $\bar{D}$ and $\bar{E}$ are the constants denoting the amplitudes of the vibrations in the x,θ and z directions, ϕ_1 and ϕ_2 are the displacement fields for higher order deformation theories for a cylindrical shell, $\phi(x)$ is the axial function that satisfies the geometric boundary conditions, a_i is the position of the *i*th ring support, P denotes the number of ring supports, ξ_i is a parameter having a value of 1 when a ring support exists and 0 when there is no ring support, n denotes the number of circumferential waves in the mode shape and ω is the natural angular frequency of the vibration. For The displacement fields defined in Eq. (61) only the transverse displacement is restrained on a ring support.

The axial function $\phi(x)$ is chosen as the beam function as [12]:

$$\begin{aligned} \phi(x) &= \gamma_1\cosh(\frac{\lambda_m x}{L}) + \gamma_2\cos(\frac{\lambda_m x}{L}) \\ &\quad -\zeta_m(\gamma_3\sinh(\frac{\lambda_m x}{L}) + \gamma_4\sin(\frac{\lambda_m x}{L})) \end{aligned} \tag{62}$$

where $\gamma_i\,(i=1,...,4)$ are some constants with value 0 or 1 chosen according to the boundary condition. λ_m, are the roots of some transcendental equations and ζ_m are some parameters dependent on λ_m. The $\gamma_i\,(i=1,...,4)$, the transcendental equations and the parameters ζ_m for the six boundary conditions are considered [12].

The geometric boundary conditions for clamped, free and simply supported boundary conditions can be expressed mathematically in terms of $\phi(x)$ as:

Clamped boundary condition

$$\phi(0) = \phi'(L) = 0 \tag{63}$$

Free boundary condition

$$\phi''(0) = \phi'''(L) = 0 \tag{64}$$

Simply support boundary condition

$$\phi(0) = \phi''(L) = 0 \tag{65}$$

Substituting Eq. (61) into Eqs. (51)-(55) and substituting Eq. (61) into Eqs. (56)-(60), for the third-order theory and the first-order theory can be expressed in matrix form as

$$[C]\begin{Bmatrix} u_1 \\ u_2 \\ u_3 \\ \phi_1 \\ \phi_2 \end{Bmatrix} - \omega^2[M]\begin{Bmatrix} \bar{A} \\ \bar{B} \\ \bar{C} \\ \bar{D} \\ \bar{E} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix} \tag{66}$$

The eigenvalue equations are solved by imposing the non-trivial solution condition and equating the determinant of the characteristic matrix $\det(C_{ij} - M_{ij}\,\omega^2) = 0$ to zero. Expanding this determinant, a polynomial in even powers of ω is obtained

$$\beta_\circ\omega^{10} + \beta_1\omega^8 + \beta_2\omega^6 + \beta_3\omega^4 + \beta_4\omega^2 + \beta_5 = \circ \tag{67}$$

where $\beta_i\,(i = 0,1,2,3,4,5)$ are some constants. Eq. (67) is solved and five positive and five negative roots are obtained.

The five positive roots obtained are the natural angular frequencies of the cylindrical shell FGM with ring support based third-order theory and first-order theory. The smallest of the five roots is the natural angular frequency studied in the present study.

9. Results and discussion

To validate the present analysis, results for simply supported-simply support and clamped-clamped FG cylindrical shells are compared with Loy and Reddy [20] and with Najafizadeh and Isvandzibaei [17]. The comparisons show that the present results agree well with those in the literature.

The functional graded cylindrical shell is composed of nickel on its inner surface and stainless steel on its outer surface. The material properties for stainless steel and nickel, calculated at $T = 300K$, are presented in Table 3.

Table 1. Comparison of natural frequency (Hz) for a FG cylindrical shell with SS-SS boundary condition.
$L = 20.3\,cm,\ R = 5.08\,cm,\ h = 0.25\,cm,\ E = 2.07788*10^{11}\,N\,m^{-2},\ \nu = 0.317756,\ \rho = 8166\,kg\,m^{-3}$

n	m	Reddy [20]	Najafizadeh & Isvandzibaei [17]	third order	first order
2	1	2050.7	2043.6	2043.0	2042.9
	2	5643.3	5635.2	5634.6	5634.2
	3	8941.3	8932.1	8932.5	8931.8
	4	11416.9	11407.2	11407.7	11407.6
	5	13262.9	13252.8	13253.5	13252.2
	6	14799.6	14789.8	14795.6	14794.3
3	1	2204.0	2195.0	2195.9	2194.4
	2	4052.0	4035.3	4038.7	4036.4
	3	6633.3	6614.3	6617.5	6615.6
	4	9140.6	9120.7	9131.8	9130.3
	5	11378.8	11358.7	11357.6	11354.6
	6	13411.9	13392.1	13389.5	13388.7

Table 2. Comparison of natural frequencies (Hz) for FG cylindrical shell with clamped-clamped (C-C) boundary condition.
$(m = 1,\ L/R = 20,\ h/R = 0.002)$

m	n	Ref. [21]	third order	first order
1	1	0.0342	0.0340	0.0337
	2	0.0119	0.0117	0.0115
	3	0.0072	0.0070	0.0078
	4	0.0089	0.0082	0.0085
	5	0.0136	0.0140	0.0148
2	1	0.0847	0.0844	0.0839
	2	0.0314	0.0317	0.0315
	3	0.0158	0.0150	0.0161
	4	0.0121	0.0122	0.0119
	5	0.0145	0.0148	0.0140

Table 3. Properties of materials.

Coefficients	Stainless steel			Nickel		
	$E\ (Nm^{-2})$	ν	$\rho(kgm^{-3})$	$E\ (Nm^{-2})$	ν	$\rho(kgm^{-3})$
P_0	201.04×10^9	0.3262	8166	223.95×10^9	0.3100	8900
P_{-1}	0	0	0	0	0	0
P_1	3.079×10^{-4}	-2.002×10^{-4}	0	-2.794×10^{-4}	0	0
P_2	-6.534×10^{-7}	3.797×10^{-7}	0	-3.998×10^{-9}	0	0
P_3	0	0	0	0	0	0
	2.07788×10^{11}	0.317756	8166	2.05098×10^{11}	0.3100	8900

where, $P_\circ, P_{-1}, P_1, P_2$ and P_3 are the coefficients of temperature $T(K)$ expressed in Kelvin and are unique to the constituent materials. The material properties P of FGMs are a function of the material properties and volume fractions of the constituent material. From the comparisons presented in Tables 1-2, it can be seen that the present analysis is accurate as the results obtained with the present analysis agreed well with those in the literature. In this paper, studies are presented for a functional graded cylindrical shell which is supported by a ring arbitrarily placed along the axial direction of the shell. This is

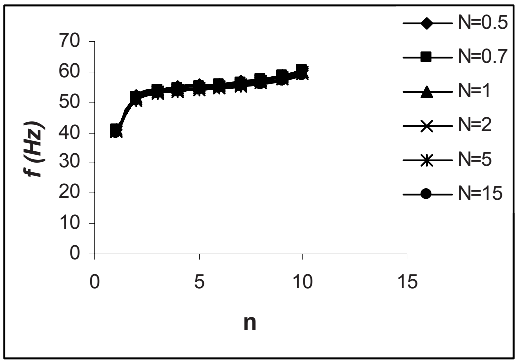

Fig. 3. Variation of the natural frequencies (Hz) against circumferential wave number n for a FG cylindrical shell with a ring support with the different volume fraction N under SS-SS boundary condition (m=1, h/R=0.01, L/R=20, a/L = 0.3).

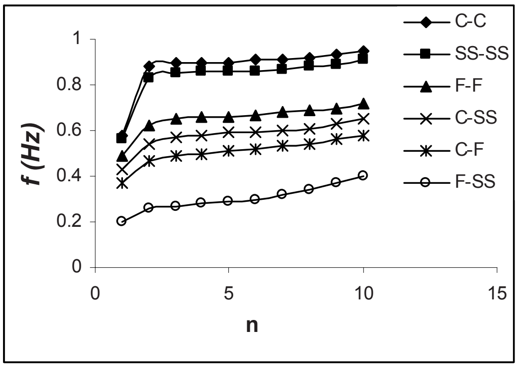

Fig. 4. Variation of the natural frequencies (Hz) with the circumferential wave number n for a functionally graded cylindrical shell with a ring support. (m = 1, h / R=0.01, L / R=20, a / L= 0.3).

carried out by setting $\xi_i = 1$ in Eq. (61). Altogether six boundary conditions, simply supported-simply supported *SS-SS*, clamped-clamped *C-C*, free-free *F-F*, clamped-simply supported *C-SS*, clamped-free *C-F* and free-simply supported *F-SS* boundary conditions, are considered in the study.

Fig. 3 shows the variation of the natural frequency (Hz) with the position of the ring support a/L for a FG cylindrical shell. The position of the ring support has a significant influence on the natural frequency and its influence varied with the volume fraction N.

Fig. 4 shows the variation of the natural frequency with the circumferential wave number n for a functional graded cylindrical shell with a ring support at $a = 0.3L$. The frequencies for the six boundary conditions increased with the circumferential wave number. This increase in frequencies is most significant when n increased from 1 to 2 and for n greater than 2 the frequencies increase gradually with the circumferential wave number. This frequency behavior indicates that the lowest frequency for a functional graded cylindrical shell with a ring support occurs at $n = 1$.

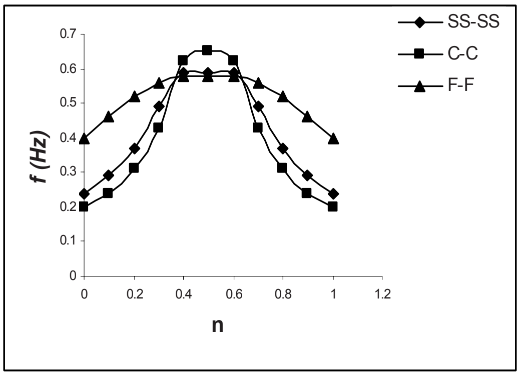

Fig. 5. Variation of the natural frequencies (Hz) versus the position of the ring support a/L for SS-SS, C-C and F-F boundary conditions (m = 1, n = 1, h/R=0.01, L/R=20).

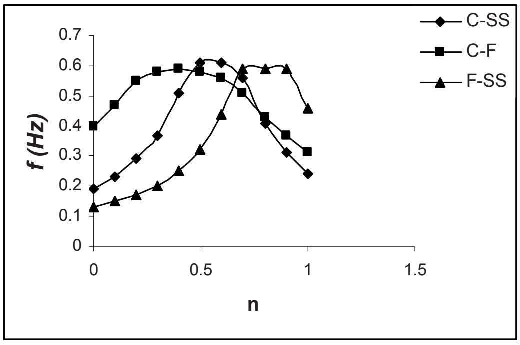

Fig. 6. Variation of the natural frequencies (Hz) versus the position of the ring support a/L for C-SS, C-F and F-SS boundary conditions (m = 1, n = 1, h/R=0.01, L/R=20).

Figs. 5 and 6 show the variation of the natural frequencies with position of the ring support. The position of the ring support has a significant influence on the natural frequencies and this influence varied with the boundary conditions. For a functionally graded cylindrical shell with ring support with same end-conditions applied in both edges, such SS-SS, C-C and F-F boundary conditions, the natural frequencies are the greatest when the ring support is in the middle of the functionally graded cylindrical shell. The natural frequencies decreased as the ring support moved away from center towards either end of the shell. Thus the natural frequencies curve is symmetrical about the center of the shell, see Fig. 5. This symmetry of the frequency curve is as expected since the end conditions are symmetrical about the ring support. For a functionally graded cylindrical shell with ring support with different end-conditions, such as C-SS, C-F and F-SS boundary conditions, the natural frequencies curve are not symmetrical about the center of the shell, see Fig. 6.

Figs. 7 and 12 show the variation of the natural frequencies FGM shell with position of the ring support a/L at different L/R ratios for the six boundary conditions. From the figures, the influence of the ring support position on the natural frequencies is generally significant at large L/R ratio. It can be seen that boundary conditions have some effects on this influence. For example in Fig. 7 the natural frequencies difference between a/L=0 and 0.5 at L/R=2 is 62.3% and L/R=10 is 154.6% while in Fig. 8 the natural frequencies difference between a/L=0 and 0.5 at L/R=2 is 51.5% and L/R=10 is 251.2%.

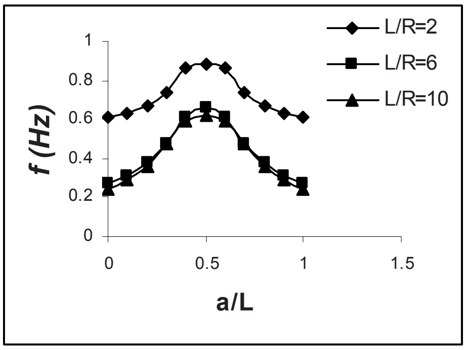

Fig. 7. Variation of the natural frequencies (Hz) with the position of the ring support a/L at different L/R ratios for SS-SS boundary conditions. (m=1, n=1, h/R=0.01)

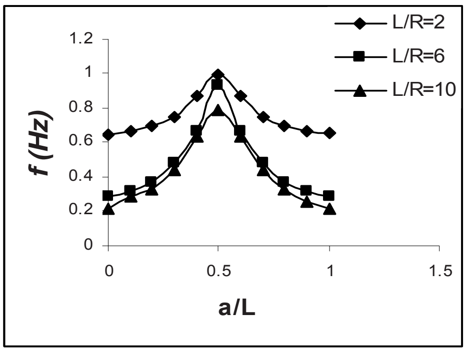

Fig. 8. Variation of the natural frequencies (Hz) with the position of the ring support a/L at different L/R ratios for C-C boundary conditions. (m=1, n=1, h/R=0.01)

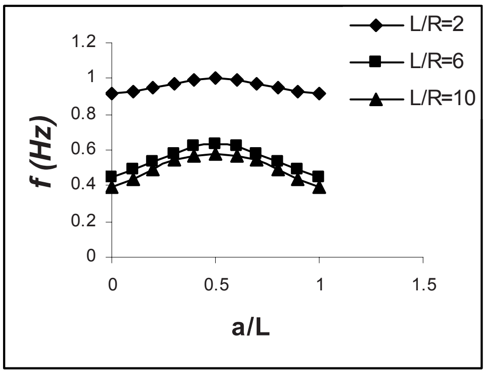

Fig. 9. Variation of the natural frequencies (Hz) with the position of the ring support a/L at different L/R ratios for F-F boundary conditions. (m=1, n=1, h/R=0.01)

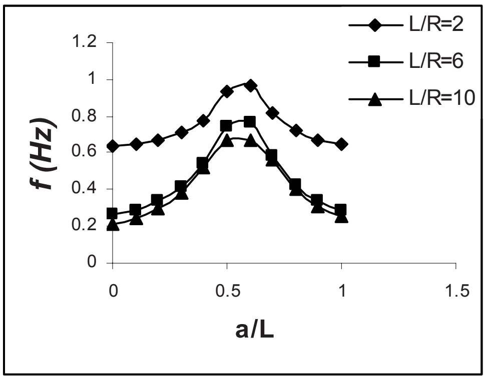

Fig. 10. Variation of the natural frequencies (Hz) with the position of the ring support a/L at different L/R ratios for C-SS boundary conditions. (m=1, n=1, h/R=0.01)

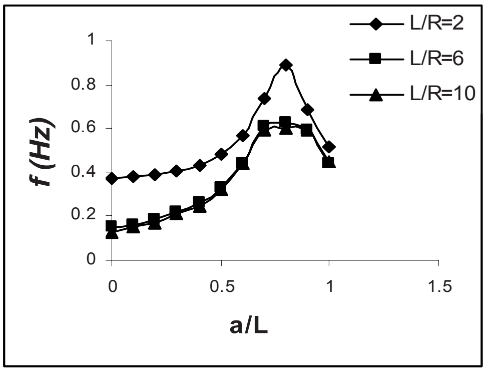

Fig. 11. Variation of the natural frequencies (Hz) with the position of the ring support a/L at different L/R ratios for C-F boundary conditions. (m=1, n=1, h/R=0.01)

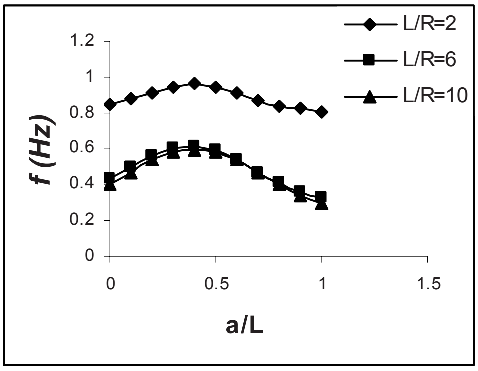

Fig. 12. Variation of the natural frequencies (Hz) with the position of the ring support a/L at different L/R ratios for F-SS boundary conditions. (m=1, n=1, h/R=0.01)

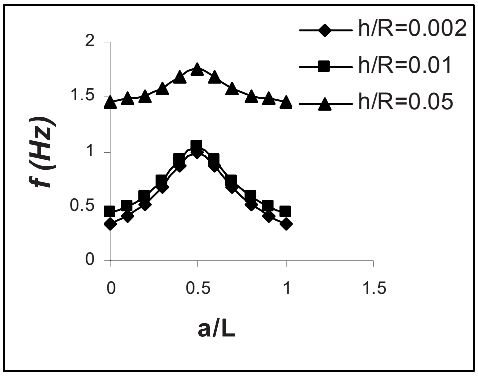

Fig. 13. Variation of the natural frequencies (Hz) with the position of the ring support a/L at different h/R ratios for SS-SS boundary conditions. (m=1, n=10, L/R=20)

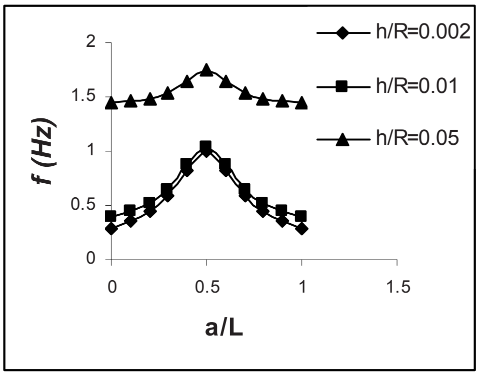

Fig. 14. Variation of the natural frequencies (Hz) with the position of the ring support a/L at different h/R ratios for C-C boundary conditions. (m=1, n=10, L/R=20)

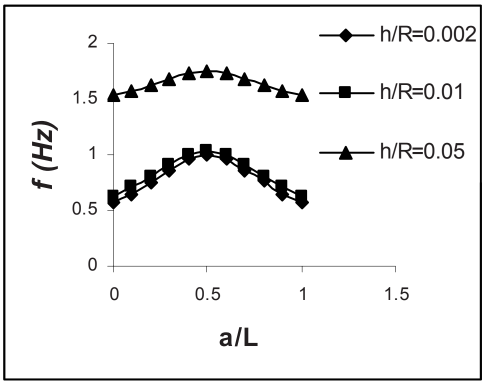

Fig. 15. Variation of the natural frequencies (Hz) with the position of the ring support a/L at different h/R ratios for F-F boundary conditions. (m=1, n=10, L/R=20)

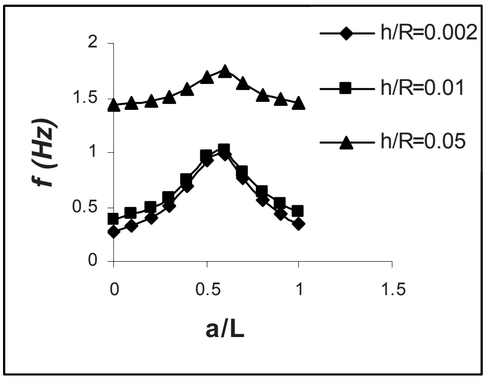

Fig. 16. Variation of the natural frequencies (Hz) with the position of the ring support a/L at different h/R ratios for C-SS boundary conditions. (m=1, n=10, L/R=20).

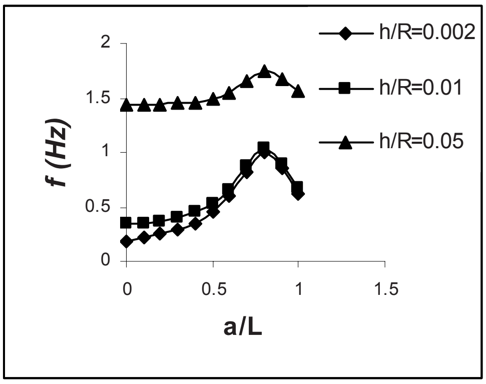

Fig. 17. Variation of the natural frequencies (Hz) with the position of the ring support a/L at different h/R ratios for C-F boundary conditions. (m=1, n=10, L/R=20)

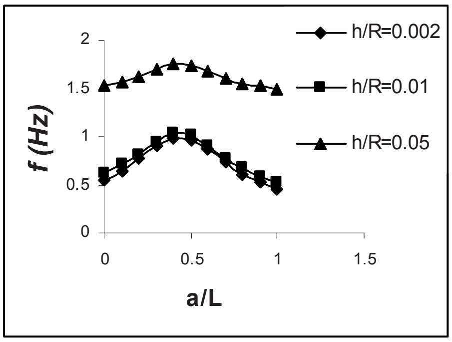

Fig. 18. Variation of the natural frequencies (Hz) with the position of the ring support a/L at different h/R ratios for F-SS boundary conditions. (m=1, n=10, L/R=20)

Figs. 13 and 18 show the variation of the natural frequencies FG shell with position of the ring support a/L at different h/R ratios for the six boundary conditions. From the figures it is apparent that the frequencies are higher at larger h/R ratios. The influence of the ring support position is significant at small h/R ratios. The frequencies are also higher at large h/R ratios.

10. Conclusions

A study on the vibration of functionally graded (FG) cylindrical shell with a ring support arbitrarily placed along the shell composed of stainless steel and nickel has been presented. Material properties are graded in the thickness direction of the shell according to volume fraction power law distribution. The study is carried out using different shear deformation shell theories with Hamilton's principle. Studies are carried out for cylindrical shells with simply supported–simply supported SS–SS, clamped –clamped C-C, free–free F–F, clamped–simply supported C–SS, clamped–free C-F and free–simply supported F–SS boundary conditions with an arbitrarily ring support along the axial direction of the cylindrical shell.

Studies were made on the frequency characteristics, the influence of ring support position and the influence of boundary conditions. The study showed that a ring support has significant influence on the frequencies and the extent of this influence depends on the position of the ring support and the boundary conditions of the functionally graded cylindrical shell. The study shows that the frequency characteristics of the functionally graded cylindrical shells are similar to homogeneous isotropic cylindrical shells. However, this is because the functionally graded cylindrical shells exhibit interesting frequency characteristics when the constituent volume fractions are varied. This is done by varying the power law exponent N .

The study showed that for a functionally graded cylindrical shell with ring support with same end-conditions applied in both edges, such SS-SS, C-C and F-F boundary conditions, the natural frequencies are the greatest when the ring support is in the middle of the functionally graded cylindrical shell and natural frequencies decreased as the ring support moved away from center towards either end of the shell, Thus the natural frequencies curve is symmetrical about the center of the shell, This symmetry of the frequency curve is as expected since the end conditions are symmetrical about the ring support, but for a functionally graded cylindrical shell with ring support with different end-conditions, such as C-SS, C-F and F-SS boundary conditions, the natural frequencies curve is not symmetrical about the center of the shell. The present analysis is validated by comparing results with those available in the literature.

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